

Which of the Following calculations gives the largest answer?

A) $1-2+3+4$ B) $1+2-3+4$ C) $1+2+3-4$

D) $1+2-3-4$ E) $1-2-3+4$

We have

$$A = 6$$

$$B = 4$$

$$C = 2$$

$$D = -4$$

$$E = 0$$

So the answer is **A**

It has just turned 22:22. How many minutes are there until midnight?

There are 38 minutes From 22:22 to 23:00

There are 60 minutes From 23:00 to midnight

So the answer is $38 + 60 = 98$

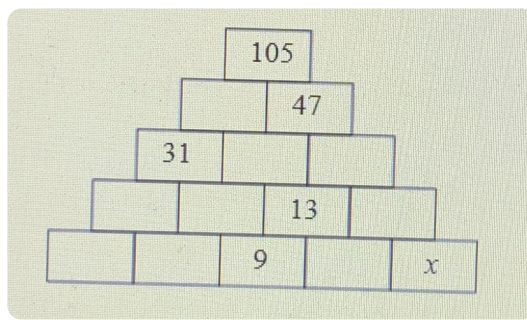
What is the value of

$$\frac{12345}{1+2+3+4+5} \quad ?$$

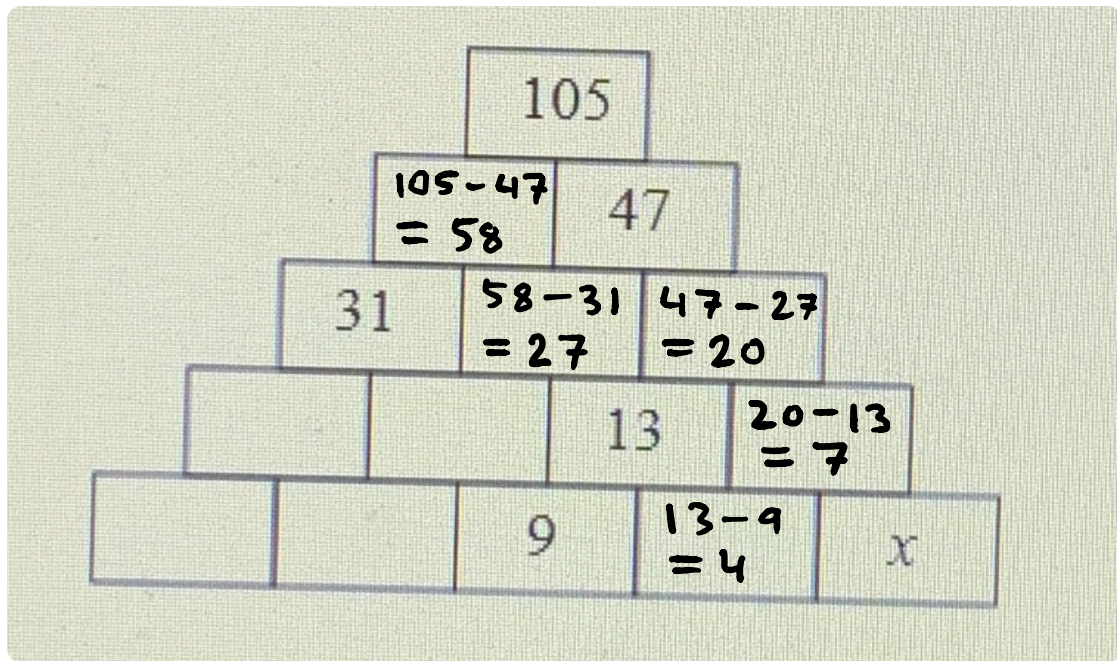
We have $1+\dots+5 = \frac{5 \times 6}{2} = 15$ and

$$12345 \div 15 = 823$$

In this partly completed pyramid, each rectangle is to be Filled with the sum of the two numbers in the rectangles immediately below it. What number should replace x ?



Fill in the spaces:



Now we have $x + 4 = 7 \Rightarrow x = 3$

The difference between $\frac{1}{3}$ of a certain number and $\frac{1}{4}$ of the same number is 3. What is that number?

Let the number be x .

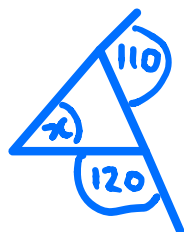
We have

$$\frac{1}{3}x - \frac{1}{4}x = 3$$

$$\Rightarrow 4x - 3x = 36$$

$$\Rightarrow x = 36$$

What is the value of x in this triangle?



By supplementary angles, the other angles of the triangle are

$$180 - 110 = 70$$

$$\text{and } 180 - 120 = 60$$

So by angle sum of a triangle

$$x = 180 - 70 - 60 = 50^\circ$$

The result of the calculation 123456789×8 is almost the same as 987654321 except that two of the digits are in a different order. What is the sum of these two digits?

The units digit of 123456789×8 is the same as the units digit of 9×8 , which is 2.

So the 2 and the 1 must have swapped places, and hence the solution is $1 + 2 = 3$

Which of the Following has the same remainder when it is divided by 2 as when it is divided by 3?

A) 3 B) 5 C) 7 D) 9 E) 11

n	n mod 2	n mod 3
3	1	0
5	1	2
7	1	1
9	1	0
11	1	2

So the answer is 7

According to a newspaper report, "A 63-year-old man has rowed around the world without leaving his living room". He clocked up 25048 miles on a rowing machine that he received for his 50th birthday. Roughly how many miles per year has he rowed since he was given the machine?

He has had the machine for 13 years and rowed roughly 26000 miles, meaning that he rowed approximately $26000 \div 13 = 2000$ miles per year

In the expression

1 $\square$ 2 $\square$ 3 $\square$ 4

each $\square$ is to be replaced by either + or $\times$. What is the largest value of all the expressions that can be obtained in this way?

The last two $\square$ should be $\times$ but multiplying by 1 doesn't increase the value of the expression so the first box should be +.

The value obtained is

$$1 + 2 \times 3 \times 4 = 1 + 24 = 25$$

What is the smallest prime number that is the sum of three different prime numbers?

Since all primes except 2 are odd, the

total must be expressed as a sum of 3 odd primes.

The smallest odd primes are 3, 5, 7, 11...

We have :

$$3 + 5 + 7 = 15 \quad \text{not prime}$$

$$3 + 5 + 11 = 19 \quad \text{prime}$$

A Fish weighs a total of 2kg plus a third of its own weight. What does it weigh?

Suppose the Fish weighs x kg.

Then

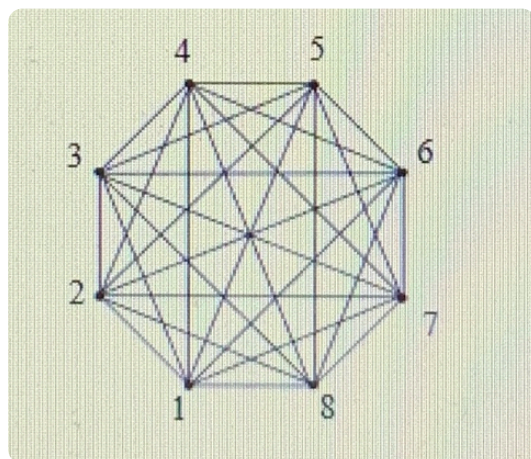
$$x = 2 + \frac{x}{3}$$

$$\Rightarrow \frac{2x}{3} = 2$$

$$\Rightarrow x = 3 \text{ kg}$$

In the Figure shown, each line joining two numbers is to be labelled with the sum of the two numbers that are at its endpoints.

How many of these labels are multiples of 3?



List all such connections, taking care not to count twice:

1+2, 1+5, 1+8

2+4, 2+7

3+6

4+5, 4+8

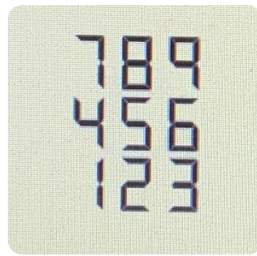
5+7

7+8

So the answer is 10

Digits on a calculator are represented by a number of horizontal and vertical illuminated bars, as shown.

How many digits are both prime and represented by a prime number of illuminated bars?



The primes in this range are 2, 3, 5, 7

2 uses 5 bars so this works.

3 uses 5 bars so this works.

5 uses 5 bars so this works.

7 uses 3 bars so this works.

So the answer is 4

Which of the Following is divisible by all of the integers from 1 to 10 inclusive?

A) 23×34 B) 34×45 C) 45×56 D) 56×67

E) 67×78

A, B and E are not divisible by 7.

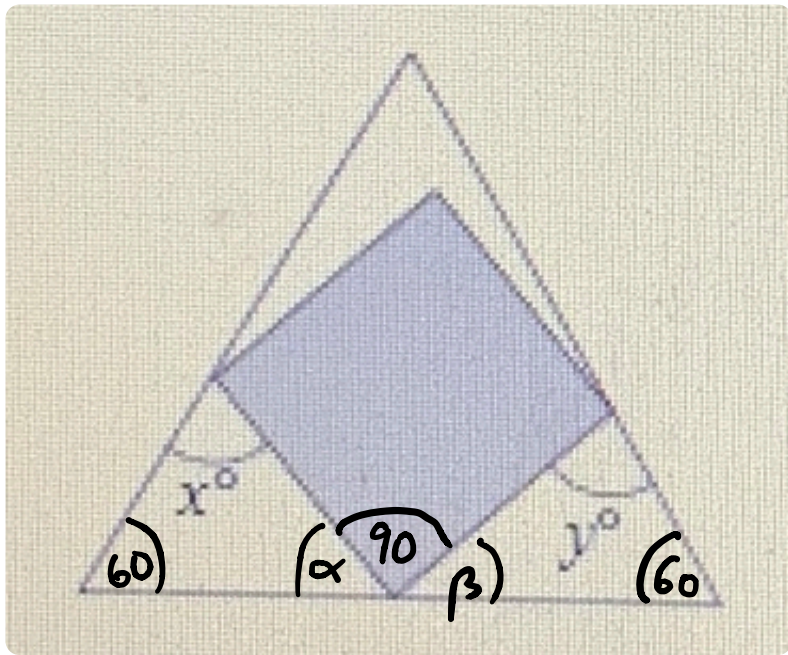
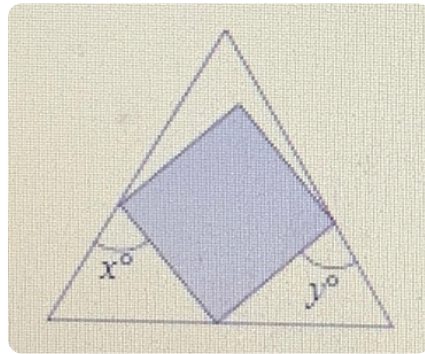
D is not divisible by 5.

For C, we have

$$\begin{aligned} 45 \times 56 &= 5 \times 9 \times 7 \times 8 \\ &= 2^3 \times 3^2 \times 5 \times 7 \end{aligned}$$

Satisfies the conditions. So the answer is **C**

The diagram shows a square inside an equilateral triangle. What is the value of $x+y$?



Label the angles as shown in the diagram.
By supplementary angles:

$$\alpha + \beta = 90^\circ$$

By angle sum of a triangle:

$$x = 120 - \alpha$$

$$y = 120 - \beta$$

$$\begin{aligned} \text{So } x+y &= 120 - \alpha + 120 - \beta \\ &= 240 - (\alpha + \beta) \\ &= 240 - 90 \\ &= \mathbf{150^\circ} \end{aligned}$$

Knave of Hearts: "I stole the tarts"

Knave of Clubs: "The Knave of Hearts is lying"

Knave of Diamonds: "The Knave of Clubs is lying"

Knave of Spades: "The Knave of Diamonds is lying"

How many of the four knaves were telling the truth?

We proceed by cases.

Suppose that Hearts is lying. Then Clubs is telling the truth, so Diamonds is lying, so Spades is telling the truth.

Suppose that Hearts is telling the truth. Then Clubs is lying, so Diamonds is telling the truth, so Spades is lying.

In both cases, **2** knaves are telling the truth.

Each of the Fractions $\frac{2637}{18459}$ and $\frac{5274}{36918}$ uses

the digits 1 to 9 exactly once. The First Fraction simplifies to $\frac{1}{7}$. What is the simplified Form of the second Fraction?

We have

$$2637 \times 2 = 5274$$

$$\text{and } 18459 \times 2 = 36918$$

so the simplified Form of the second Fraction is also **$\frac{1}{7}$**

One of the Following cubes is the smallest cube that can be written as the sum of three positive cubes. Which is it?

A) 27 B) 64 C) 125 D) 216 E) 512

List the First Few cubes:

$$1^3 = 1$$

$$2^3 = 8$$

$$3^3 = 27$$

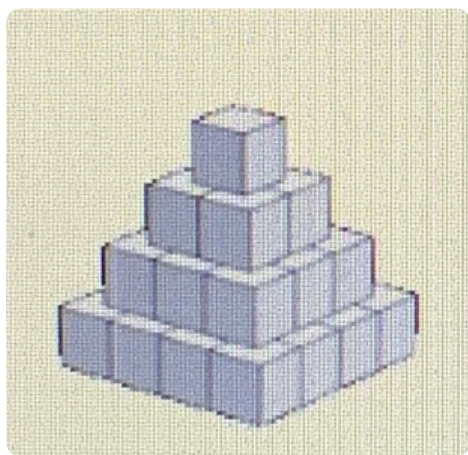
$$4^3 = 64$$

$$5^3 = 125$$

$$6^3 = 216$$

We have $216 = 125 + 64 + 27$ and none of the lower cubes can be written as a sum of three others. So the answer is **D**

The diagram shows a pyramid made up of 30 cubes, each measuring $1\text{m} \times 1\text{m} \times 1\text{m}$. What is the total surface area of the whole pyramid, including its base?



The area of the base is $4 \times 4 = 16$

The total area of the tops is also 16 since if we look at the pyramid from above, we can see a 4 by 4 square.

The area of the sides is

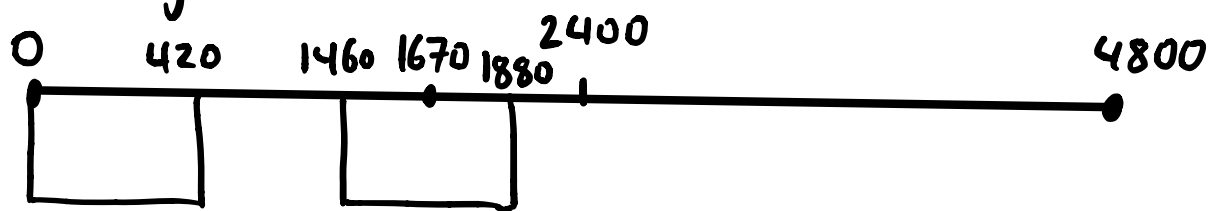
$$4 \times (4 + 3 + 2 + 1) = 4 \times 10 = 40$$

So the total surface area is

$$16 + 16 + 40 = 72 \text{ m}^2$$

Gill has 4 pictures to hang in a horizontal row on a wall which is 4800mm wide. The pictures are identical in size and are 420mm wide. Gill hangs the First two pictures so that one is on the extreme left of the wall and one is on the extreme right of the wall. She wants to hang the remaining two pictures so that all four pictures are equally spaced. How far should Gill place the centre of each of the two remaining pictures From a vertical line down the centre of the wall?

Draw a diagram:



We only need to consider one half of the wall since it is symmetrical about the line down the centre (at 2400mm).

The paintings take up a length of

$$420 \times 4 = 1680 \text{ mm}$$

So the remaining space is $4800 - 1680 = 3120 \text{ mm}$

Since the paintings are equally spaced, the space between two adjacent paintings is

$$\frac{3120}{3} = 1040 \text{ mm}$$

So the second painting starts at

$$420 + 1040 = 1460 \text{ mm}$$

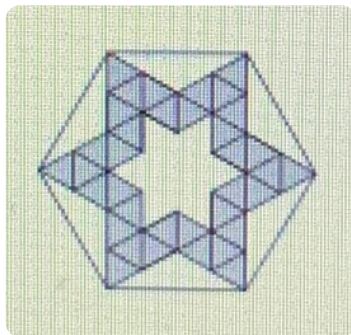
and has centre point

$$1460 + 210 = 1670 \text{ mm}$$

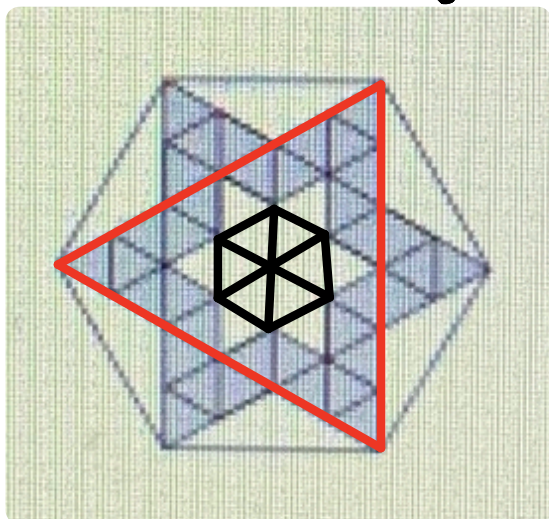
Hence the distance between this point and the centre of the wall is

$$2400 - 1670 = 730 \text{ mm}$$

The diagram shows a shaded region inside a regular hexagon. The shaded region is divided into equilateral triangles. What fraction of the area of the hexagon is shaded?



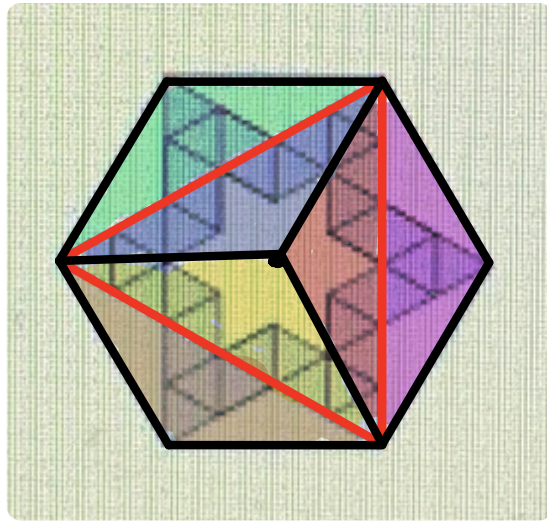
Divide the hexagon into sections as shown:



There are 12 unshaded triangles inside the red triangle, and 12 shaded triangles outside it.

If we rearranged those triangles, the question reduces to Finding the Fraction of the total area inside the red triangle.

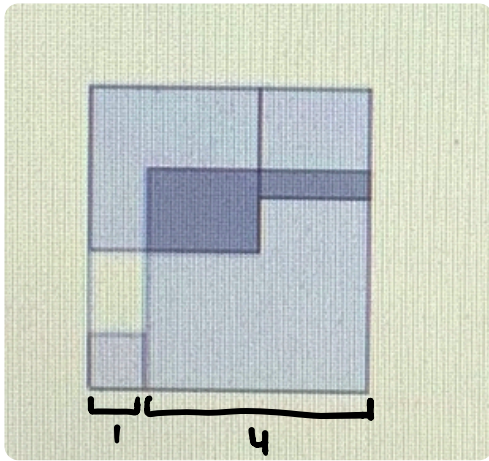
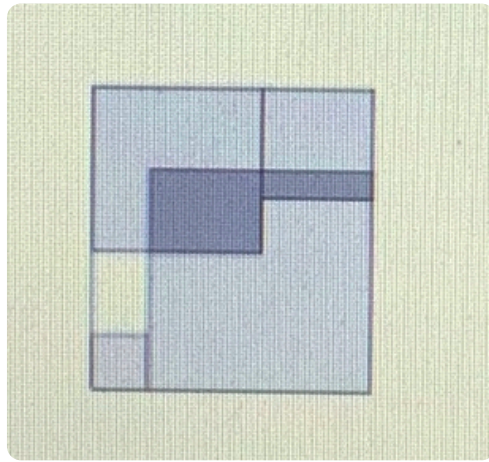
Consider the Following division of the hexagon into 6 congruent triangles :



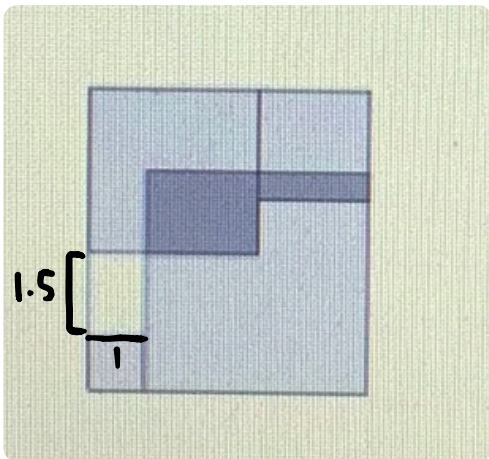
This shows that the area inside the red triangle is half of the total area.

So the answer is $\frac{1}{2}$

The diagram shows four shaded glass squares, with areas 1cm^2 , 4cm^2 , 9cm^2 , 16cm^2 placed in the corners of a rectangle. The largest square overlaps two others. The area of the region inside the rectangle but not covered by any square is 1.5cm^2 . What is the area of the region where the squares overlap?



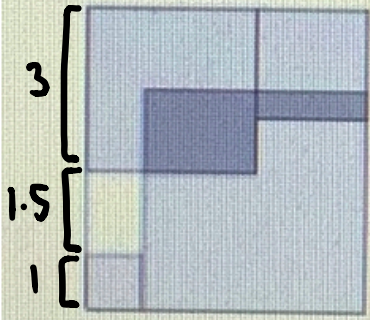
The width of the rectangle is
 $1 + 4 = 5$



The unshaded rectangle has width 1 and area 1.5 so its height must be 1.5

The height of the rectangle is

$$3 + 1.5 + 1 = 5.5$$



Let the area of the shaded region be x .

We have

$$\underbrace{5 \times 5.5}_{\text{Area of rectangle}} = \underbrace{1 + 4 + 9 + 16}_{\text{Areas of the squares}} + \underbrace{1.5}_{\text{unshaded area}} - \underbrace{x}_{\text{subtract the overlap as it was counted twice}}$$

$$\Rightarrow 27.5 = 31.5 - x$$

$$\Rightarrow x = 4 \text{ cm}^2$$

A palindromic number is a number that reads the same when the order of its digits is reversed. What is the difference between the largest and smallest 5-digit palindromic numbers that are both multiples of 45?

A multiple of 45 is a multiple of both 5 and 9. So it must end in a 5 (can't end in 0 since it can't start with a 0) and the sum of its digits must be a multiple of 9.

Since the number ends in 5, it must also start with 5.

To maximise the value, the thousands digit should be a 9. So the tens digit will be also.

The sum of the digits is currently

$$5 + 9 + 9 + 5 = 28$$

so the hundreds digit should be 8 in order to obtain a total of $9 \times 4 = 36$.

To minimise the value, the thousands digit should be a 0. So the tens digit will be also.

The sum of the digits is currently

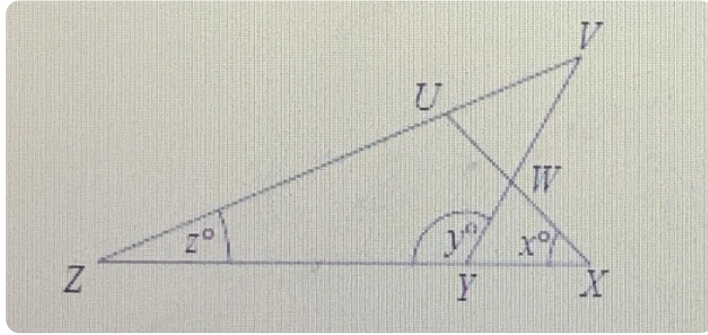
$$5 + 0 + 0 + 5 = 10$$

so the hundreds digit should be 8 in order to obtain a total of $9 \times 2 = 18$.

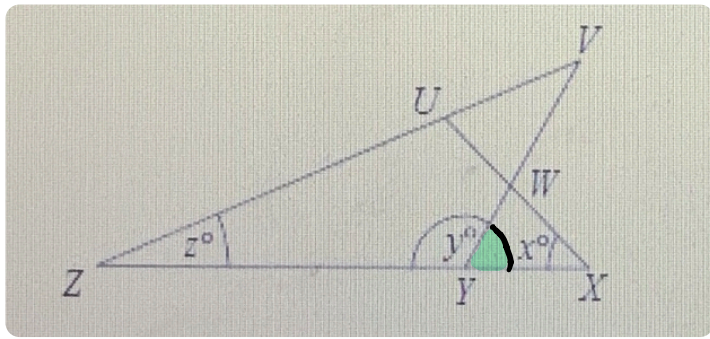
Hence the answer is

$$59895 - 50805 = 9090$$

The Four straight lines in the diagram are such that $VU = VW$. The sizes of $\angle Uxz$, $\angle VYz$ and $\angle Vzx$ are x , y , z degrees. Which of the following equations gives x in terms of y and z ?

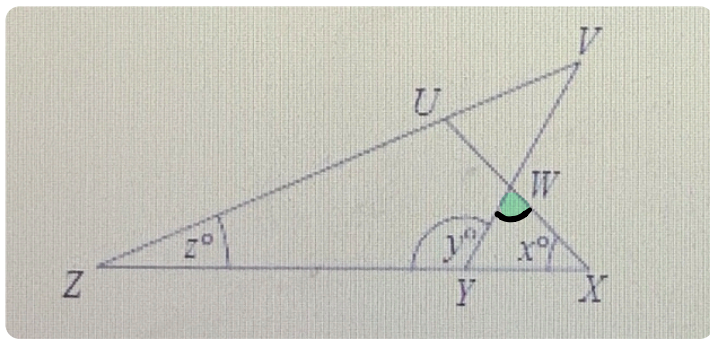


- A) $x = y - z$ B) $x = 180 - y - z$ C) $x = y - \frac{z}{2}$
 D) $x = y + z - 90$ E) $x = \frac{y - z}{2}$



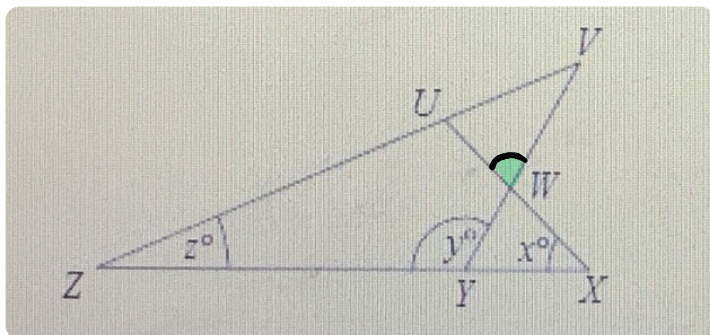
By supplementary angles

$$\angle WYX = 180 - y$$



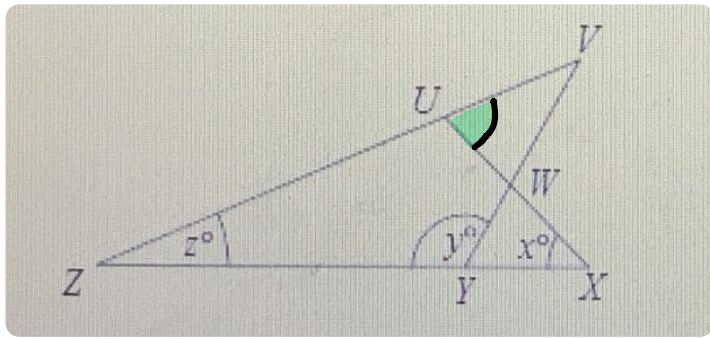
By angle sum of a triangle

$$\begin{aligned}\angle YWX &= 180 - x - (180 - y) \\ &= y - x\end{aligned}$$



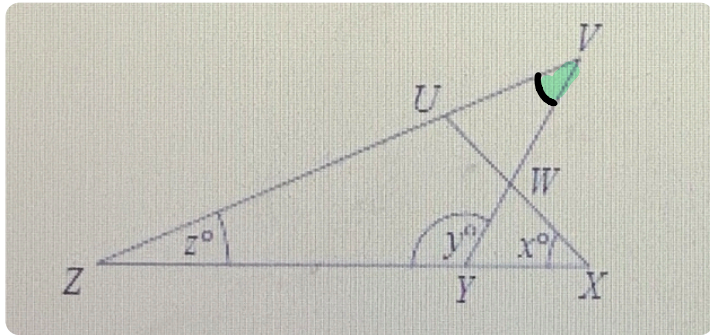
By vertically opposite angles

$$\begin{aligned}\angle UWV &= \angle YWX \\ &= y - x\end{aligned}$$

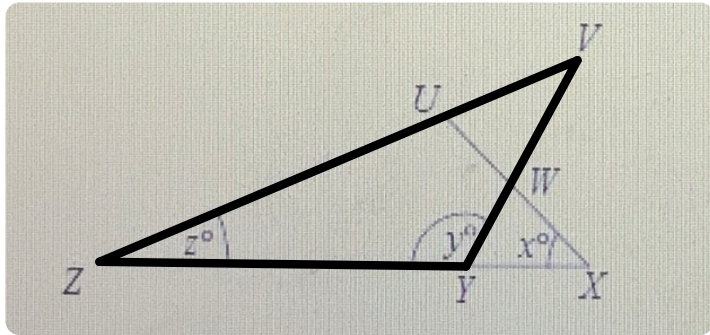


Since $VU = VW$

$$\begin{aligned}\angle VUW &= \angle UWV \\ &= y - x\end{aligned}$$



By angle sum of a triangle
 $\angle UVW = 180 - 2(y - x)$



By angle sum of a triangle

$$\begin{aligned}180 &= z + y + (180 - 2(y - x)) \\ &= z + y + 180 - 2y + 2x \\ \Rightarrow 0 &= z - y + 2x\end{aligned}$$

$$\Rightarrow 2x = y - z$$

$$\Rightarrow x = \frac{y - z}{2}$$